Boson Journal of Modern Physics (BJMP)
ISSN: 2454-8413

SCITECH
RESEARCH ORGANISATION|

Volume 2, Issue 1

October 23, 2015|

Boson Journal of Modern Physics

www.scitecresearch.com

One-Loop Contribution to the Matter-Driven Expansion of the Universe

Bogusław Broda
Department of Theoretical Physics, Faculty of Physics and Applied Informatics, University of Łódź, 90-236 Łódź, Pomorska 149/153, Poland.

Abstract

Standard perturbative quantum gravity formalism is applied to compute the lowest order corrections to the spatially flat cosmological FLRW solution governed by ordinary matter. The presented approach is analogous to the one used to compute quantum corrections to the Coulomb potential in electrodynamics, or to the approach applied in computation of quantum corrections to the Schwarzschild solution in gravity. In this framework, it is shown that the corrections to the classical metric coming from the one-loop graviton vacuum polarization (self-energy) have (UV cutoff dependent) repulsive properties, which could be not negligible in the very early Universe.

Keywords: One-loop graviton vacuum polarization; One-loop graviton self-energy; Quantum corrections to classical gravitational fields; Early Universe; Quantum cosmology.

1. Introduction

The aim of our work it to explicitly show the appearance of "repulsive forces" of quantum origin, which could be not negligible in the very early evolution of the Universe. Actually, we apply the method used earlier in the case of the spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) solution governed by radiation [1] (see also [2,3]). It appears that the cosmological FLRW case with ordinary matter as a source is similar to the radiation one. Namely, the lowest order quantum corrections coming from the fluctuating graviton vacuum are "repulsive", resembling the situation well-known in loop quantum cosmology (LQC) [4-6]. The phenomenon is obviously negligible in our epoch, but it could be not so in the very early Universe. One should stress that the derivation is a lowest order approximation—the graviton vacuum polarization (self-energy) is taken in one-loop approximation, and the approach assumes the validity of the weak-field regime.

2. One-Loop Corrections

Our starting point is a general spatially flat FLRW space-time

$$ds^2 = g_{\mu\nu}\,dx^\mu dx^\nu = dt^2 - a^2(t)\mathbf{dr}^2, \tag{1}$$

with the cosmic time-dependent scale factor $a(t)$. To satisfy the condition of weakness of the gravitational field $\kappa h_{\mu\nu}$ near the reference time $t = t_0$ in the perturbative expansion

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \tag{2}$$

($\kappa = \sqrt{32\pi G_N}$, with the Newtonian gravitational constant G_N), the metric is rescaled in such a way that it is exactly Minkowski one for $t = t_0$, i.e.

$$a^2(t) = 1 - \kappa h(t), \qquad h(t_0) = 0. \tag{3}$$

Then

$$h_{\mu\nu}(t,\mathbf{r}) = h(t)\mathcal{I}_{\mu\nu} \quad \text{and} \quad \mathcal{I}_{\mu\nu} \equiv \begin{pmatrix} 0 & 0 \\ 0 & \delta_{ij} \end{pmatrix}. \tag{4}$$

In view of the standard harmonic gauge condition (see the second eq. in (8)), which we impose, we perform the following gauge transformation:

$$\kappa h_{\mu\nu} \to \kappa h'_{\mu\nu} = \kappa h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \quad \text{with} \quad \xi_\mu(t) = \left(-\frac{3\kappa}{2}\int_0^t h(t')\,dt',\, 0,\, 0,\, 0\right). \tag{5}$$

For simplicity, skipping the prime, we get

$$h_{\mu\nu}(t,\mathbf{r}) = h(t)\begin{pmatrix} -3 & 0 \\ 0 & \delta_{ij} \end{pmatrix} \quad \text{and} \quad h_\lambda^\lambda(t) = -6h(t), \tag{6}$$

where the indices are being manipulated with the flat Minkowski metric $\eta_{\mu\nu}$. Switching from $h_{\mu\nu}$ to standard ("better") perturbative gravitational variables, namely to the "barred" field $\bar{h}_{\mu\nu}$ which is defined by

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - 1/2\,\eta_{\mu\nu} h_\lambda^\lambda, \tag{7}$$

we get

$$\bar{h}_{\mu\nu}(t,\mathbf{r}) = -2h(t)\mathcal{I}_{\mu\nu} \quad \text{with} \quad \partial^\mu \bar{h}_{\mu\nu} = 0. \tag{8}$$

The Fourier transform of $\bar{h}_{\mu\nu}$ is of the form

$$\tilde{\bar{h}}_{\mu\nu}(p) = -2\tilde{h}(E)(2\pi)^3\delta^3(\mathbf{p})\mathcal{I}_{\mu\nu}. \tag{9}$$

To obtain quantum corrections to classical field we should supplement the classical line with a vacuum polarization (self-energy) contribution and a corresponding (full) propagator. Therefore, the lowest order quantum corrections $\tilde{\bar{h}}^{\mathrm{q}}{}_{\mu\nu}$ to the classical gravitational field $\tilde{\bar{h}}^{\mathrm{c}}{}_{\mu\nu}$ are given, in the momentum representation, by the formula (see, e.g. [7] , or §114 in [8] for an electrodynamic version—the so-called Uehling potential)

$$\tilde{\bar{h}}^{\mathrm{q}}{}_{\mu\nu}(p) = \left(D\Pi\tilde{\bar{h}}^{\mathrm{c}}\right)_{\mu\nu}(p), \tag{10}$$

Where

$$D_{\mu\nu}^{\alpha\beta}(p) = \frac{i}{p^2}\mathbb{D}_{\mu\nu}^{\alpha\beta} \tag{11}$$

is the free graviton propagator in the harmonic gauge with the auxiliary (constant) tensor $\mathbb{D}$ defined below in Eq.(12), and $\Pi_{\mu\nu}^{\alpha\beta}(p)$ is the (one-loop) graviton vacuum polarization (self-energy) tensor operator. Now, we define the auxiliary tensors:

$$\mathbb{D} \equiv \mathbb{E} - 2\mathbb{P}, \quad \text{where} \quad \mathbb{E}_{\mu\nu}^{\alpha\beta} \equiv 1/2\left(\delta_\mu^\alpha\delta_\nu^\beta + \delta_\nu^\alpha\delta_\mu^\beta\right) \quad \text{and} \quad \mathbb{P}_{\mu\nu}^{\alpha\beta} \equiv 1/4\,\eta^{\alpha\beta}\eta_{\mu\nu}; \tag{12}$$

which satisfy the following useful identities:

$$\mathbb{E}^2 = \mathbb{E}, \quad \mathbb{P}^2 = \mathbb{P}, \quad \mathbb{E}\mathbb{P} = \mathbb{P}\mathbb{E} = \mathbb{P} \quad \text{and} \quad \mathbb{D}^2 = \mathbb{E}. \tag{13}$$

By virtue of the definition (7) we find that

$$\bar{h}_{\mu\nu} = (\mathbb{D}h)_{\mu\nu}. \tag{14}$$

Multiplying Eq.(10) from the left by $\mathbb{D}$, we get (using (11), (14), and the last identity in (13))

$$\tilde{h}^{\mathrm{q}}{}_{\mu\nu}(p) = \frac{i}{p^2}\left(\Pi\tilde{\bar{h}}^{\mathrm{c}}\right)_{\mu\nu}(p). \tag{15}$$

Actually, a substantial simplification takes place in Eq.(15), namely,

$$\tilde{h}^{\mathrm{q}}{}_{\mu\nu}(p) = \frac{i}{p^2}\left(\Pi'\tilde{\bar{h}}^{\mathrm{c}}\right)_{\mu\nu}(p), \tag{16}$$

where $\Pi'(p)$ is an "essential" part of the full (in the sense of the one-loop approximation) graviton polarization operator $\Pi(p)$. The "essential" part $\Pi'(p)$ of the full (one-loop) graviton vacuum polarization operator $\Pi(p)$ can be obtained from $\Pi(p)$ by skipping all the terms with the momenta p with free indices (e.g. α, β, μ, or ν). Such a simplification follows from the gauge freedom the $\tilde{h}^{\mathrm{q}}{}_{\mu\nu}$ enjoys, and from the harmonic gauge condition the $\tilde{\bar{h}}^{\mathrm{c}}{}_{\alpha\beta}$ should satisfy. In general, by virtue of the symmetry of the indices, $\Pi(p)$ consists of 5 (tensor) terms. Each p_μ can be ignored in $\Pi(p)$ because it only gives rise to a gauge transformation of $\tilde{h}_{\mu\nu}$. Moreover, since $\tilde{\bar{h}}_{\alpha\beta}$ satisfies the harmonic gauge condition, the terms with p^α in $\Pi(p)$ are annihilated. In other words, schematically

$$\Pi(p) = \Pi'(p) + \cdots p \cdots . \tag{17}$$

Since the momenta p in the ellipses posses free indices, they can be ignored, and only the first two terms with dummy indices (i.e. p^2) survive, i.e.

$$\Pi'(p) = \kappa^2 p^4 I(p^2)(2\alpha_1 \mathbb{E} + 4\alpha_2 \mathbb{P}), \tag{18}$$

where the numerical values of the coefficients α_1 and α_2 depend on the kind of the field circulating in the loop, and the (scalar) standard loop integral $I(p^2)$ with the UV cutoff denoted by M is asymptotically of the form (see, e.g., Chapt. 9.4.2 in [9])

$$I(p^2) = \frac{1}{(2\pi)^4} \int \frac{d^4 q}{q^2 (p-q)^2} = -\frac{i}{(4\pi)^2} \log\left(-\frac{p^2}{M^2}\right) + \cdots \quad , \tag{19}$$

where the dots mean terms $\mathcal{O}(p^2/M^2)$. A standard way to derive (19) consists in continuing from q_0 to $+iq_4$ ($d^4 q \to i d^4 q_{\mathrm{E}}$), exponentiating the denominator using a (double) proper-time representation for the propagators, a change of proper-time variables, imposing the UV cutoff for a new proper time, and next continuing back to the Minkowski momentum variables. Thus, we obtain

$$\widetilde{h^{\mathrm{q}}}_{\mu\nu}(p) = \frac{i}{p^2} \kappa^2 p^4 \left[-\frac{i}{(4\pi)^2} \log\left(-\frac{p^2}{M^2} \right) \right] \left[-2\widetilde{h^{\mathrm{c}}}(E)(2\pi)^3 \delta^3(\mathbf{p}) \right] [(2\alpha_1 \mathbb{E} + 4\alpha_2 \mathbb{P}) \mathcal{I}]_{\mu\nu}$$

$$= -2\pi\kappa^2 E^2 \log\left|\frac{E}{M}\right| \widetilde{h^{\mathrm{c}}}(E) \delta^3(\mathbf{p}) \begin{pmatrix} -3\alpha_2 & 0 \\ 0 & (2\alpha_1 + 3\alpha_2)\delta_{ij} \end{pmatrix}. \tag{20}$$

3. Matter Source

Now, we specify our input classical metric. To this end, we choose the matter source assuming

$$a^2(t) = \left|\frac{t}{t_0}\right|^{4/3}. \tag{21}$$

According to (3) and (21) the Fourier transform of $h^{\mathrm{c}}(t)$ is

$$\widetilde{h^{\mathrm{c}}}(E) = \frac{2}{\kappa t_0^{4/3}} \sin(2\pi/3) \Gamma(7/3) |E|^{-7/3} + \cdots, \tag{22}$$

where the dots mean a term (vanishing in (20)) proportional to the Dirac delta. Performing the gauge transformation in the spirit of (5), we remove the purely time component of $h^{\mathrm{q}}{}_{\mu\nu}$, i.e. $h^{\mathrm{q}}{}_{00} \to h^{\mathrm{q}\prime}{}_{00} = 0$. The inverse Fourier transform yields now the quantum correction

$$h^{\mathrm{q}}{}_{\mu\nu}(t) = \frac{\alpha\kappa}{(3\pi)^2 t_0^{4/3}} |t|^{-2/3} (\log|t/t_c| + c) \mathcal{I}_{\mu\nu}, \tag{23}$$

where $c \equiv \gamma + \frac{3}{2}\log 3 + \frac{\pi}{3}\sqrt{3}$ (γ is the Euler–Mascheroni constant), t_c is an UV cutoff in time units, and $\alpha \equiv 2\alpha_1 + 3\alpha_2$. According to the table given in [1] only the graviton field yields a non-zero contribution with $\alpha = -1/16$. Now one can easily check that the second time derivative of (23) is positive for $t > t_c$. Therefore the quantum contribution to the classical expansion is accelerating.

4. Final Remarks

In the framework of the standard (one-loop) perturbative quantum gravity, we have derived the formula (23) expressing a contribution to the classical metric governing matter-driven expansion of the Universe. As the second time derivative of (23) is positive we expect an accelerating role of this contribution, especially in early evolution of the Universe. In spite of the fact that (23) is UV cutoff dependent, the (qualitative) result is unchanged as long as $t > t_c$.

Supported by the University of Łódź grant.

References

[1] Broda, B (2011). One-loop quantum gravity repulsion in the early Universe. Physical Review Letters, 106(10), 101303. doi:10.1103/PhysRevLett.106.101303.

[2] Broda, B (2011). Quantum gravity stability of isotropy in homogeneous cosmology. Physics Letters B, 704(5), 655-658. doi: 10.1016/j.physletb.2011.09.087.

[3] Broda, B (2014). Perturbative quantum damping of cosmological expansion. Physics Letters B, 732, 182-185. doi: 10.1016/j.physletb.2014.03.048.

[4] Ashtekar, A & Sloan, D (2009). Loop quantum cosmology and slow roll inflation. Physics Letters B, 694, 108-112. doi: 10.1016/j.physletb.2010.09.058.

[5] Ashtekar, A, Bojowald, M & Lewandowski, J (2003). Mathematical structure of loop quantum cosmology. Advances in Theoretical and Mathematical Physics, 7(2), 233-268. doi: 10.4310/ATMP.2003.v7.n2.a2.

[6] Bojowald, M. (2008). Loop Quantum Cosmology. Living Reviews in Relativity, 11(4), 1-131.

[7] Duff, MJ (1974). Quantum corrections to the Schwarzschild solution. Physical Review D, 9(6), 1837-1839. doi:10.1103/PhysRevD.9.1837.

[8] Berestetskii, VB, Pitaevskii, LP & Lifshitz, EM (1982). Quantum Electrodynamics, vol. 4, 2^{nd} ed.Butterworth-Heinemann.

[9] Huang, K (1992). Quarks, Leptons and Gauge Fields, 2^{nd} ed. World Scientific Publishing Company.